International Journal of Thermal Sciences 48 (2009) 1943–1948

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Mixed convection boundary layer flow past an isothermal horizontal circular cylinder with temperature-dependent viscosity

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ARTICLE INFO

Article history:
Received 18 July 2008
Received in revised form
19 February 2009
Accepted 19 February 2009
Available online 18 March 2009

Keywords:
Mixed convection
Boundary layer
Horizontal circular cylinder
Temperature-dependent viscosity

ABSTRACT

The problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder placed in a viscous and incompressible fluid of temperature-dependent viscosity is theoretically considered in this paper. The partial differential equations governing the flow and heat transfer are shown to be non-similar. Full numerical solutions of these governing equations are obtained using an implicit finite-difference scheme known as the Keller-box method. The solutions are obtained for various values of the Prandtl number *Pr*, the mixed convection parameter λ and the viscosity/temperature parameter θ_r. The obtained results show that the flow and heat transfer characteristics are significantly influenced by these parameters.

1. Introduction

Mixed or combined convective heat transfer in boundary layer flow represents an important problem, which is frequently encountered in many industrial and technical processes including solar central receivers exposed to winds, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment, etc. [1]. Further, the typical physical example of variable fluid properties can be found in the oil cooling of the electronic equipment [2]. In this kind of convective flows, the free and forced convection effects are of comparable magnitude [3]. It is very well-known that in several practical applications, there exist significant temperature differences between the surface of the hot body and the free stream. These temperature differences cause density gradients in the fluid medium and in the presence of a gravitational body force, free convection effects become important. When natural or mixed convection heat transfer takes place under conditions where there are large temperature differences within the fluid, it is necessary (for accuracy) to consider the effects of variable fluid properties. The typical physical example can be found in the oil cooling of the electronic equipment.

Studies on mixed convection boundary layer flow past a horizontal circular cylinder have been conducted previously by several researchers. It appears that Sparrow and Lee [4] were the first to study the problem of mixed convection boundary layer flow about a horizontal circular cylinder. Later, Merkin [5] has studied the problem of mixed convection from a horizontal circular cylinder and he solved it based on the Crank–Nicolson method, using Newton–Raphson method along with Choleski decomposition technique. Further, Nazar et al. [6–9] have solved similar problems for Newtonian as well as for micropolar fluids and they have considered the cases of constant wall temperature and constant wall heat flux.

An interesting macroscopic physical phenomenon in fluid mechanics is the variation of viscosity with temperature. For many liquids, among them petroleum oils, glycerine, silicone fluid and some molten salts, the variation of absolute viscosity with temperature is often much greater than that of the other properties [2]. In all of the above mentioned studies on mixed convection boundary layer flow past a horizontal circular cylinder, the fluid viscosity is treated as a constant. However, it is well-known that the changes of this physical property may relate to the temperature. For example, the viscosity of water decreases by about 58% when the temperature increases from 10 °C ($\mu = 1.31 \times 10^{-3}$ kg m^{-1}s^{-1}) to 50 °C ($\mu = 5.48 \times 10^{-4}$ kg m^{-1}s^{-1}). In order to make good predictions of the flow behaviour, it is important to take into account this variation of viscosity. It has been shown by Gary et al. [10] and

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doi:10.1016/j.ijthermalsci.2009.02.014

Nomenclature

a	radius of the cylinder
C_f	skin friction coefficient
f	non-dimensional stream function
g	acceleration due to gravity
Gr	Grashof number, $Gr = g\beta\Delta T a^3/\nu^2$
k	thermal conductivity
Nu	Nusselt number
Pr	Prandtl number
q_w	heat flux from the cylinder
Re	Reynolds number, $Re = U_\infty a/\nu$
T	local fluid temperature
T_r	reference temperature
T_w	cylinder temperature
T_∞	ambient temperature
au, v	non-dimensional velocity components along the x – and y – directions, respectively
$u_e(x)$	non-dimensional velocity outside boundary layer
U_∞	free stream velocity
x, y	non-dimensional Cartesian coordinates along the surface of the cylinder and normal to it, respectively
x_s	boundary layer separation point

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	thermal property of the fluid
θ	non-dimensional temperature
θ_r	viscosity/temperature parameter
λ	mixed convection parameter
ν	kinematic viscosity, $\nu = \mu/\rho$
ν_∞	constant kinematic viscosity of the ambient fluid
μ	dynamic viscosity
μ_∞	constant dynamic viscosity of the ambient fluid
ρ	fluid density
τ_w	wall shear stress
ψ	stream function

Subscripts

w	condition at the surface of the cylinder
∞	ambient/free stream condition

Superscripts

$'$	differentiation with respect to y
$-$	dimensional variables

Mehta and Sood [11] that when variable viscosity is taken into account, the flow characteristics may be substantially changed compared to the constant viscosity case. Lings and Dybbs [12] studied the forced convection with variable viscosity over flat plate in a porous medium. Kafoussias and Williams [1] and Kafoussias et al. [13] have investigated the effect of temperature-dependent viscosity on mixed convection boundary layer flow past a vertical isothermal flat plate. Hossain et al. [14,15] studied the natural or free convection flow about a vertical cone and vertical wavy surfaces, respectively, with the viscosity inversely proportional to the linear function of temperature. Further, Molla et al. [16] have considered a problem of natural convection flow about an isothermal horizontal circular cylinder with temperature-dependent viscosity. Recently, Ahmad et al. [17] have studied the effect of temperature-dependent viscosity on free convection over isothermal cylinders of elliptic cross section. Finally, we mention to this end the interesting paper by Ali [18] where he considered the problem of the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface.

Therefore, in order to get more accurate information about the flow and temperature characteristics, the aim of this paper is to study the problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder with the effect of temperature-dependent viscosity immersed in a viscous fluid for both assisting (heated cylinder) and opposing flow (cooled cylinder) cases. The partial and ordinary differential equations governing the flow and temperature fields are solved numerically using an efficient implicit finite-difference scheme known as the Keller-box method (see [19]). The results obtained are compared with those reported by Merkin [5] for a constant viscosity when the Prandtl number is unity, namely $Pr = 1$ and it is found that the results are in very good agreement. It should be mentioned that the method used by Merkin [5] consists in replacing derivatives of the partial differential equations in the direction by differences and all other quantities averaged. The two non-linear ordinary differential equations which result were solved by writing them in finite-difference form and solving the non-linear algebraic equations iteratively by a Newton–Raphson process. The linear algebraic equations arising in the iterative process were solved by Choleski decomposition technique. We believe that the present results are more accurate than those which use the usual assumption of constant properties [20].

2. Mathematical formulation

We consider a problem of mixed convection boundary layer flow past a horizontal circular cylinder of radius a placed in a viscous and incompressible fluid of temperature-dependent viscosity. It is also assumed that the cylinder is kept at the uniform temperature T_w, while the ambient fluid has the constant temperature T_∞, where $T_w > T_\infty$ (heated cylinder) corresponds to an assisting flow (free stream and buoyancy forces are in the same direction where the buoyancy forces will assist the fluid to accelerate in the boundary layer) and $T_w < T_\infty$ (cooled cylinder) corresponds to an opposing flow (free stream and buoyancy forces are in the opposite directions where the buoyancy forces will retard the fluid flows in the boundary layer). Further, following Merkin [5] it is assumed that the characteristic velocity is $(1/2)U_\infty$. Under these assumptions along with the Boussinesq and boundary layer approximations, the basic boundary layer equations of this problem are:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho \bar{u}_e(\bar{x})\frac{\mathrm{d}\bar{u}_e(\bar{x})}{\mathrm{d}\bar{x}} + \frac{\partial}{\partial \bar{y}}\left(\mu \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \rho g \beta (T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subject to the boundary conditions

$$\bar{u} = \bar{v} = 0,\ T = T_w \ \text{at}\ \bar{y} = 0, \quad \bar{u} \to \bar{u}_e(\bar{x}),\ T \to T_\infty \ \text{as}\ \bar{y} \to \infty, \tag{4}$$

where $\bar{x}$ is the coordinate measured along the surface of the cylinder starting from the lower stagnation point ($\bar{x} \approx 0$) and $\bar{y}$ is the distance measured normal to it, $(\bar{u}, \bar{v})$ are the velocity components along the $(\bar{x}, \bar{y})$ axes, T is the local fluid temperature and the other physical quantities are defined in the Nomenclature. Further, $(\bar{x}/a)$ is the angle of the cylinder as shown in Fig. 1.

We assume that the dynamic viscosity μ has the form proposed by Lings and Dybbs [12]

$$\mu = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)}, \tag{5}$$

so that the viscosity is an inverse linear function of temperature T. Here μ_∞ is a constant dynamic viscosity of the ambient fluid and γ is a thermal property of the fluid, which is a constant. Equations (1)–(3) can be non-dimensionalised using the following new variables:

$$x = \bar{x}/a, \;\; y = Re^{1/2}(\bar{y}/a), u = \bar{u}/U_\infty, \;\; v = Re^{1/2}(\bar{v}/U_\infty),$$
$$\theta = (T - T_\infty)/(T_w - T_\infty), \;\; u_e(x) = \bar{u}_e(\bar{x})/U_\infty, \tag{6}$$

where $(Re = U_\infty a/\nu)$ is the Reynolds number.

The non-dimensional temperature θ can also be written as

$$\theta = \frac{T - T_r}{T_w - T_\infty} + \theta_r, \tag{7}$$

where T_r is a constant and its value depends on the reference state and the constant γ. Air and water are the most common fluids involved in engineering applications. Therefore, Kafoussias and Williams [1] have shown the appropriateness of Eq. (5) by giving the correlations between viscosity and temperature for air and water as follows:

for air,

$$\frac{1}{\mu} = -123.2(T - 742.6), \tag{8}$$

based on $T_\infty = 293K(20\,^\circ\mathrm{C})$ and for water,

$$\frac{1}{\mu} = 29.83(T - 258.6), \tag{9}$$

based on $T_\infty = 288K(15\,^\circ\mathrm{C})$.

The viscosity/temperature parameter θ_r in Eq. (7) is given by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)} = \text{constant}, \tag{10}$$

and its value is determined by the viscosity/temperature characteristics of the fluid and the operating temperature difference $\Delta T = T_w - T_\infty$. It is worth mentioning that in the case when the temperature difference ΔT is positive, θ_r must physically be >1 for gases and <0 for liquids. However, the opposite is true if ΔT is negative, where θ_r must physically be >1 for liquids and <0 for gases since γ has the opposite sign in each of these cases and vice versa [21]. Using (5)–(7), the basic boundary layer equations (1)–(3) can be written in non-dimensional form as

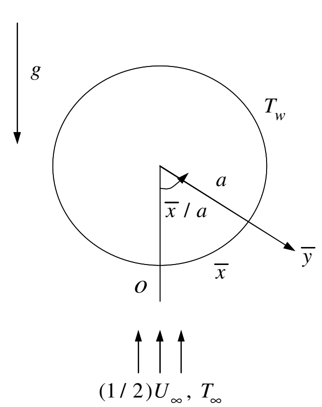

Fig. 1. Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e(x)\frac{\mathrm{d}u_e(x)}{\mathrm{d}x} + \frac{\theta_r}{(\theta - \theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} + \frac{\theta_r}{(\theta_r - \theta)}\frac{\partial^2 u}{\partial y^2} + \lambda\theta \sin x, \tag{12}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{13}$$

and the boundary conditions (4) become:

$$u = v = 0, \;\; \theta = 1 \text{ at } y = 0, \;\; u \to u_e(x), \;\; \theta \to 0 \text{ as } y \to \infty, \tag{14}$$

where Pr is the Prandtl number and λ is the mixed convection parameter which describes the relative importance of free convection to forced convection and is defined as

$$\lambda = \frac{Gr}{Re^2}, \tag{15}$$

and $(Gr = g\beta\Delta T a^3/\nu^2)$ is the Grashof number. It should be mentioned that $\lambda > 0$ corresponds to assisting flow, $\lambda < 0$ corresponds to opposing flow and $\lambda = 0$ corresponds to forced convection flow.

3. Solution procedure

We take $u_e(x) = \sin x$ [5] where $u_e(x)$ is the potential flow (outer flow) for a circular cylinder in the present dimensionless variables. In order to solve Eqs. (11)–(13), subject to the boundary conditions (14), we assume the following variables:

$$\psi = xf(x, y), \;\; \theta = \theta(x, y), \tag{16}$$

where ψ is the stream function defined in the usual way as:

$$u = \frac{\partial \psi}{\partial y}, \;\; v = -\frac{\partial \psi}{\partial x}. \tag{17}$$

Substituting (16) and (17) into Eqs. (12) and (13), we get, after some algebra, the resulting equations:

$$\frac{\partial^3 f}{\partial y^3} - \frac{\theta - \theta_r}{\theta_r} f\frac{\partial^2 f}{\partial y^2} + \frac{\theta - \theta_r}{\theta_r}\left(\frac{\partial f}{\partial y}\right)^2 - \frac{1}{(\theta - \theta_r)}\frac{\partial \theta}{\partial y}\frac{\partial^2 f}{\partial y^2} - \frac{\sin x \cos x}{x}\frac{(\theta - \theta_r)}{\theta_r} - \lambda\frac{\sin x}{x}(\theta - \theta_r)\frac{\theta}{\theta_r} = x\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2} - \frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y}\right)\frac{\theta - \theta_r}{\theta_r}, \tag{18}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{19}$$

subject to the boundary conditions (14) which become:

$$f = \frac{\partial f}{\partial y} = 0, \;\; \theta = 1 \text{ at } y = 0,$$
$$\frac{\partial f}{\partial y} \to \frac{\sin x}{x}, \;\; \theta \to 0 \text{ as } y \to \infty. \tag{20}$$

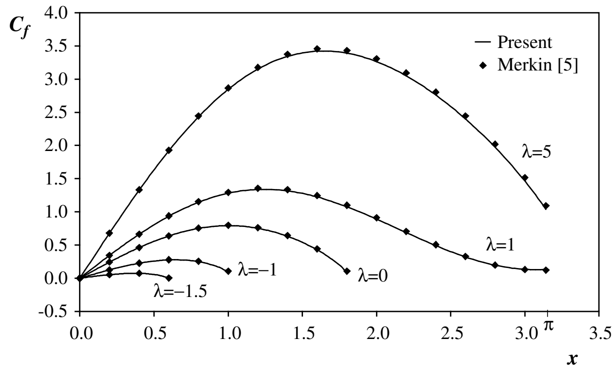

Fig. 2. The skin friction coefficient C_f for various values of λ when $Pr = 1$ (case of constant viscosity).

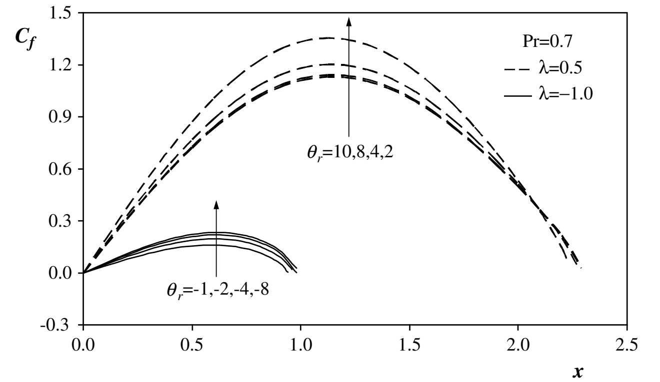

Fig. 4. The skin friction coefficient C_f for various values of θ_r when $Pr = 0.7$ and $\lambda = 0.5$ (assisting flow), $\lambda = -1.0$ (opposing flow).

The quantities of interest are the skin friction coefficient C_f and the Nusselt number Nu which are defined as

$$C_f = \frac{aRe^{-1/2}}{\mu_\infty U_\infty}\tau_w, \quad Nu = \frac{aRe^{-1/2}}{k(T_w - T_\infty)}q_w, \tag{21}$$

where τ_w and q_w are given by

$$\tau_w = \left(\mu\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{22}$$

with k being the thermal conductivity of the fluid. Using the non-dimensional variables (6) and (22) into Eq. (21), we get

$$C_f = \frac{x\theta_r}{\theta_r - 1}\frac{\partial^2 f}{\partial y^2}(x,0), \quad Nu = -\frac{\partial \theta}{\partial y}(x,0). \tag{23}$$

It can be seen that at the lower stagnation point of the cylinder, i.e. ($x \approx 0$), Eqs. (18) and (19) reduce to the following ordinary differential equations:

$$f''' - \frac{\theta - \theta_r}{\theta_r}ff'' + \frac{\theta - \theta_r}{\theta_r}f'^2 - \frac{1}{(\theta - \theta_r)}\theta' f'' - \frac{\theta - \theta_r}{\theta_r} - \lambda(\theta - \theta_r)\frac{\theta}{\theta_r} = 0, \tag{24}$$

$$\frac{1}{Pr}\theta'' + f\theta' = 0, \tag{25}$$

subject to the boundary conditions

$$f(0) = f'(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \tag{26}$$

where primes denote differentiation with respect to y.

We notice from the paper by Kafoussias and Williams [1] that for physically realizable situation, θ_r cannot take values between 0 and 1, and as mentioned before, the constraint $\theta_r > 1$ is for gases and $\theta_r < 0$ is for liquids when λ is positive and vice versa. It is also important to point out that when $|\theta_r|$ is large ($|\theta_r| \to \infty$), the viscosity variation in the boundary layer is negligible and Eqs. (18), (19), (24) and (25) reduce to those found by Merkin [5]. However, as $\theta_r > 1$ is for gases or $\theta_r < 0$ is for liquids, the viscosity variation becomes increasingly significant.

4. Results and discussion

The two sets of Eqs. (18), (19), (24) and (25) subject to the boundary conditions (20) and (26), respectively, have been solved numerically using an implicit finite-difference scheme known as the Keller-box method along with the Newton's linearization technique as described by Cebeci and Bradshaw [19]. The solution is obtained in the following four steps:

1. Reduce Eqs. (18), (19), (24) and (25) to a first-order system.
2. Write the difference equations using central differences.
3. Linearize the resulting algebraic equations by Newton's method and write them in matrix-vector form.
4. Solve the linear system by the block-tridiagonal-elimination technique.

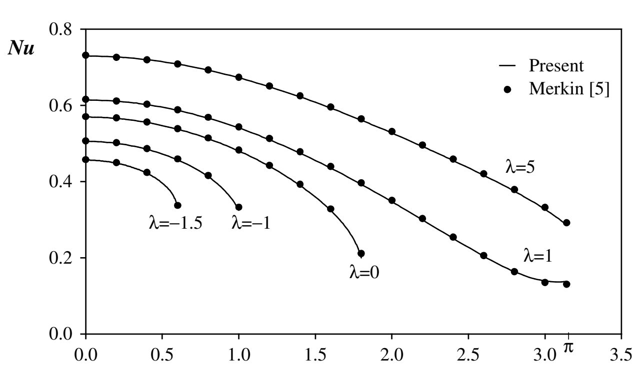

Fig. 3. The Nusselt number Nu for various values of λ when $Pr = 1$ (case of constant viscosity).

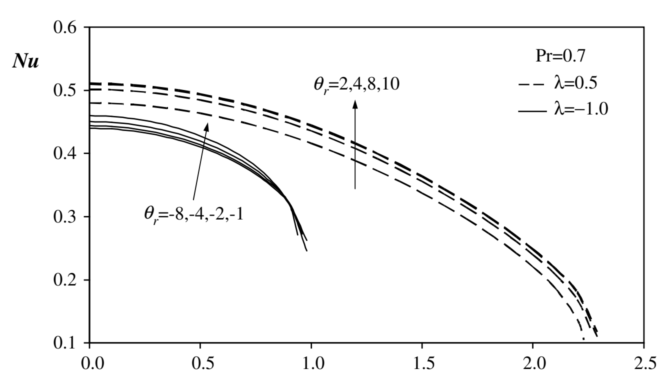

Fig. 5. The Nusselt number Nu for various values of θ_r when $Pr = 0.7$ and $\lambda = 0.5$ (assisting flow), $\lambda = -1.0$ (opposing flow).

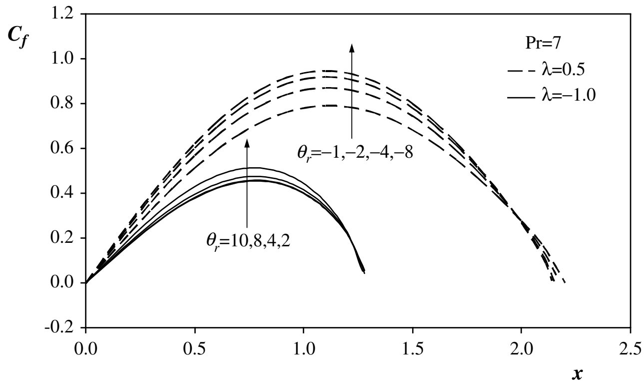

Fig. 6. The skin friction coefficient C_f for various values of θ_r when $Pr = 7$ and $\lambda = 0.5$ (assisting flow), $\lambda = -1.0$ (opposing flow).

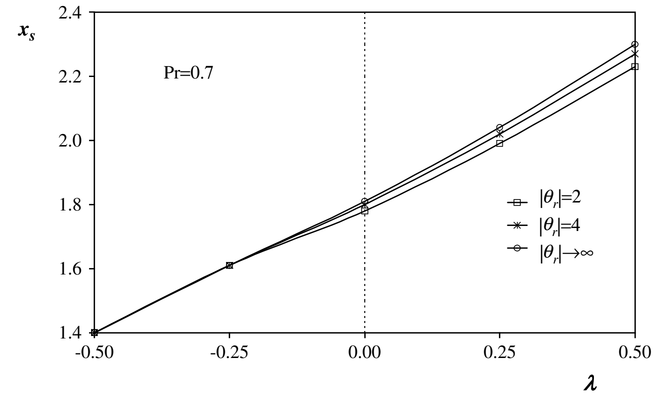

Fig. 8. Variation of the separation point x_s with λ for $Pr = 0.7$ when $|\theta_r| = 2$, 4 and $|\theta_r| \to \infty$ (constant viscosity).

The numerical solution starts at the lower stagnation point of the cylinder, $x \approx 0$, with the initial profiles as given by Eqs. (24) and (25) subject to the boundary conditions (26) and proceed around the cylinder up to the separation point x_s. In order to identify the optimum step size, the value of $\Delta y = 0.02$ was found to be satisfactory for the calculations of f and θ with a convergence criterion of 10^{-5} which gives about four decimal places accuracy (see [19]). The step size of x, Δx and the edge of the boundary layer, y_∞ are adjusted for different range of parameters. Numerical results for the skin friction coefficient C_f and the Nusselt number Nu have been obtained for various values of Prandtl number, $Pr = 1$, 0.7 (air) and 7 (water) with various values of the mixed convection parameter λ and the viscosity/temperature parameter θ_r at different positions of x around the surface of the cylinder.

It is worth mentioning that the numerical scheme used in the present study, namely the Keller-box method, has been proven to be unconditionally stable and it is also the most flexible of the common method, being easily adaptable to solving equations of any order [19]. On the other hand, in order to verify the accuracy of the present method, the values of the skin friction coefficient C_f and the Nusselt number Nu for $Pr = 1$ when $\lambda = -1.5, -1$, 0, 1 and 5 are compared with those reported by Merkin [5] in Figs. 2 and 3 by taking $|\theta_r| \to \infty$ (constant viscosity) in this study. These results are found to be in good agreement. The results also show that the separation of boundary layer is delayed as the mixed convection parameter λ increases. It is also found that the boundary layer does not separate ($x_s = \pi$) when $\lambda \geq 1$.

Numerical results for the skin friction coefficient and Nusselt number for $Pr = 0.7$ at different positions x with various values of θ_r for both assisting ($\lambda = 0.5$) and opposing ($\lambda = -1.0$) flow cases are presented in Figs. 4 and 5, respectively. It is seen from Fig. 4 that at each point of x, the skin friction coefficient increases as θ_r decreases for the opposing flow but for the assisting flow, this result only valid to a certain value of x. Meanwhile, Fig. 5 shows that at each point of x, the Nusselt number increases as θ_r increases, i.e. the increasing of θ_r promotes the heat transfer for the assisting flow but for the opposing flow, this result only valid to a certain value of x.

Figs. 6 and 7 show the numerical results for the skin friction coefficient and Nusselt number, respectively, for $Pr = 7$ at different positions x with various values of θ_r for both assisting ($\lambda = 0.5$) and opposing ($\lambda = -1.0$) flow cases. It is noticed from Fig. 6 that at each point of x, the skin friction coefficient increases as θ_r decreases for both assisting and opposing flows cases and this result only valid to a certain value of x. Further, Fig. 7 shows that for both assisting and opposing flows cases, the Nusselt number increases as θ_r increases at each point of x.

Further, Figs. 4 and 6 show that for each θ_r, there exists a maximum value of skin friction coefficient for both assisting and opposing flows cases. It is observed that the point of x where the skin friction is maximum for the assisting flow is greater than the opposing flow, for each value of Pr. It can also be seen from Figs. 4 and 6 that the skin friction coefficient for air ($Pr = 0.7$) is greater than water ($Pr = 7$) when the flow is assisting. On the other hand, the skin friction coefficient for water is greater than air when the flow is opposing. Figs. 5 and 7 show that for all values of θ_r, the Nusselt number decreases as x increases up to the separation point x_s for both assisting and opposing flows cases. Those figures also

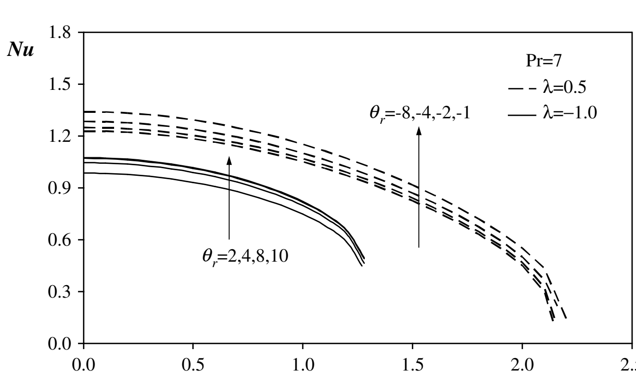

Fig. 7. The Nusselt number Nu for various values of θ_r when $Pr = 7$ and $\lambda = 0.5$ (assisting flow), $\lambda = -1.0$ (opposing flow).

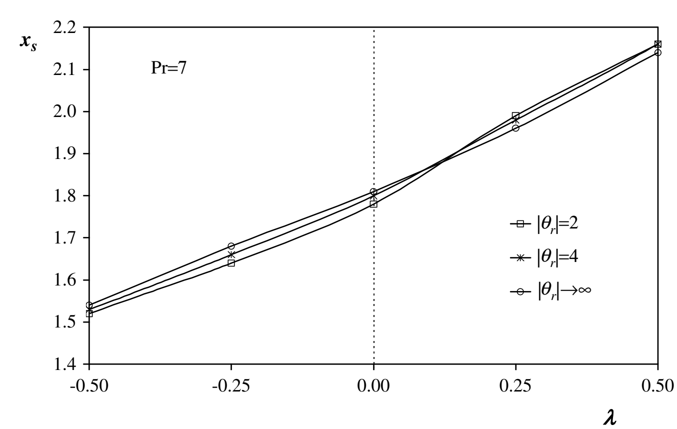

Fig. 9. Variation of the separation point x_s with λ for $Pr = 7$ when $|\theta_r| = 2$, 4 and $|\theta_r| \to \infty$ (constant viscosity).

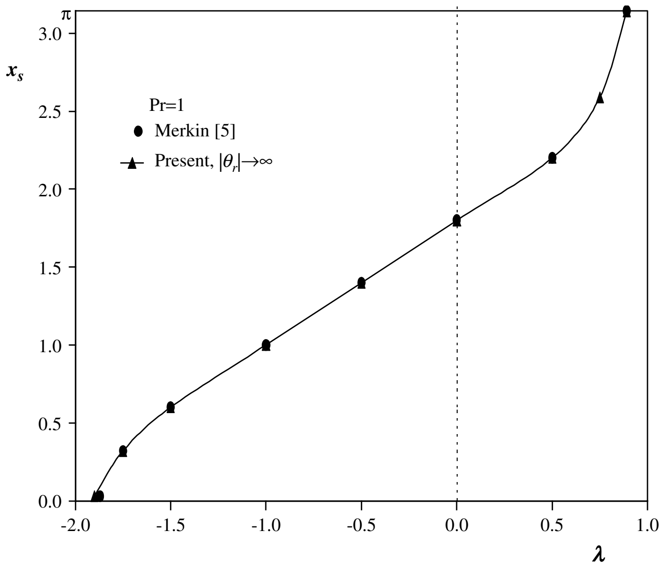

Fig. 10. Variation of the separation point x_s with λ for $Pr = 1$ when $|\theta_r| \to \infty$ (constant viscosity).

show that the Nusselt number for water is greater than air for both assisting and opposing flows. It can be seen also from Figs. 4–7 that the skin friction coefficient and Nusselt number for the assisting flow are greater than the opposing flow for each value of *Pr*.

Numerical results for the separation point x_s when $Pr = 0.7$ and 7 for both assisting and opposing flows cases are presented in Figs. 8 and 9. These two figures show that the boundary layer separation is delayed (i.e. the position of the separation point increases along the surface of the cylinder) as λ increases for each value of considered $|\theta_r|$. It is worth mentioning that these results are also coinciding with those found by Merkin [5] for $Pr = 1$ (as shown in Fig. 10). Fig. 8 also shows that for $Pr = 0.7$, increasing $|\theta_r|$ delays the boundary layer separation and this happens only when the flow is assisted. On the other hand, we can see in Fig. 9 that for $Pr = 7$, increasing $|\theta_r|$ also delays the boundary layer separation but only for the opposing flow case. Finally, it should be mentioned that in a similar way to the case of constant viscosity ($|\theta_r| \to \infty$), the present numerical results show that in those cases when the boundary layer separates, $C_f \to 0$ and $Nu \to Nu_s(\neq 0)$ as $x \to x_s$ in a singular way as was found by Merkin [5].

5. Conclusions

A numerical study is performed for the problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder placed in a viscous and incompressible fluid of temperature-dependent viscosity. Calculations are carried out for various values of the Prandtl number *Pr*, the mixed convection parameter λ and the viscosity/temperature parameter θ_r. The obtained results show that the flow and thermal characteristics is significantly influenced by these parameters. Therefore, it is important to consider the effect of temperature-dependent viscosity especially when the viscosity of a fluid is sensitive to temperature variations, otherwise considerable errors may occur in the characteristics of the heat transfer process.

Acknowledgement

The authors wish to express their very sincere thanks to the reviewers for the valuable comments and suggestions. This work is supported by the Research University Grant Scheme (RUGS) from Universiti Putra Malaysia.

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